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Factorization in B decays within soft-collinear effective theory

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Abstract. We discuss the most general form of the leading power suppressed collinear operators in the soft-collinear effective theory. Such operators appear in the description of power corrections to exclusive heavy flavor decays into energetic light hadrons. We consistently include the effects of three-particle light-cone distribution amplitudes and find that their impact could be of phenomenological relevance.

PACS. 12.39.St Factorisation – 12.39.Hg Heavy quark effective theory

1 Introduction

The soft-collinear effective theory (SCET) [1,2,3,4,5,6] has been proposed as a systematic framework for the study of processes involving energetic light quarks and gluons. Possible applications include the decays of heavy hadrons into light particles in the kinematical regions where the final products are very energetic, and hard scattering processes involving light hadrons, such as deep inelastic scattering and exclusive hadron form factors at large momentum transfer. SCET provides a natural framework for establishing a systematic expansion in Λ/Q where Λ is the QCD scale and Q is the typical large energy of the particles involved. In particular, it provides a convenient tool to establish factorization theorems and study power corrections.

In the following we use the standard light-cone decomposition of momenta

$$p^\mu = \frac{1}{2} n^\mu \bar{n}\cdot p + \frac{1}{2}\bar{n}^\mu n\cdot p + p_\perp^\mu \equiv (p_+, p_-, p_\perp)\,, \qquad (1)$$

where n and $\bar{n}$ are light-cone vectors satisfying $n^2 = \bar{n}^2 = 0$, $n\cdot\bar{n} = 2$. In any given process n and $\bar{n}$ are chosen to be aligned to the final state collinear momenta.

In the description of heavy meson decays, there are three relevant kinematical configurations.

a) Soft quarks (q_s, h_v) and gluons (A_s^μ) with momenta $p_s \simeq \Lambda = Q(\lambda,\lambda,\lambda)$ (where we defined $\lambda = \Lambda/Q$). The exchange of soft particles can only be parameterized and results in the non-factorizable contributions to heavy-to-heavy and heavy-to-light form factors and the B meson wave function.

b) Collinear quarks (ξ_n) and gluons (A_c^μ) with momenta $p_c \simeq Q(1,\lambda^2,\lambda)$. These modes appear in the description of the constituents of a fast-moving light meson and their exchanges have to be parameterized in terms of the light-cone wave functions of the final state mesons.

c) Hard collinear quarks (Ξ_n) and gluons (A_{hc}^μ) with momenta $p_{hc} \simeq Q(1,\lambda,\lambda^{1/2})$. These modes are necessary to describe interactions of soft fields with collinear particles. Since their virtuality is perturbative, they can be integrated out and result in so-called jet-functions.

It is always possible to completely integrate out hard collinear particles because in all applications they always appear as internal modes. Technically, this is realized in a two step procedure. In the first step an effective theory is formulated ($\mathrm{SCET_I}$) containing soft, collinear and hard-collinear modes, which is matched into a second step onto the final effective theory ($\mathrm{SCET_{II}}$) containing only collinear and soft modes.

For the purpose of this note, we take the point of view that problems associated with the integration over the hard collinear modes at subleading order (see, for instance, the recent analyses presented in [7,8]) have been cleared and proceed to the analysis of power suppressed contributions. Once the process is specified, the integration of the hard collinears gives, order by order in α_s, all the relevant SCET operators (that will involve only soft and collinear modes). The matrix elements of these operators between initial and final states fall in two groups: those factorizable in terms of "conventional" form factors and light-cone wave functions, and others that require the introduction of new non-perturbative objects. In particular, the light-cone wave functions enter through matrix elements of SCET operators (involving two quarks and an infinite number of gluons) between the vacuum and a meson state.

In Sect. 2 we introduce all the possible SCET collinear operators that appear at leading and subleading order in λ. In Sect. 3 we compute their matrix elements in terms of the usual light cone distribution amplitudes of pseudoscalar [9,10] and vector [11,12,13] mesons. These operators are necessary for the SCET analysis of any process involving energetic light mesons.

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2 Collinear bilinears

The operators to be constructed in this section contain the collinear quark $\xi_{n,p}$ and gluon $A_{n,q}$ fields, together with the collinear covariant derivative $iD_c^\mu = \mathcal{P}^\mu + gA_n^\mu$. In order to be able to perform the power counting of the various contributions at the operator level, it is convenient to assign a λ scaling to the fields requiring the kinetic terms to be of order $O(1)$. In this way, one obtains $\xi_n \sim \lambda$ and $A_c^\mu \sim (\lambda^2, 1, \lambda)$. In the hybrid formulation of SCET, one achieves a correct λ expansion by extracting the large Fourier modes from each field:

$$\phi_c(x) = \sum_{\tilde{p}_c} e^{-i\tilde{p}_c x}\phi_{c,\tilde{p}_c}(x) \tag{2}$$

where $\tilde{p}_c = Q(0,1,\lambda)$, are labels and the new field $\phi_{c,\tilde{p}_c}$ is responsible for the soft fluctuations. It is convenient to introduce a "label" operator $\mathcal{P}^\mu$ [3] which picks the large momentum of a collinear field: $\mathcal{P}^\mu\,\xi_{n,p} = (\frac{\bar{n}\cdot p}{2}n^\mu + p_\perp^\mu)\,\xi_{n,p}$. When acting on a product of several fields, these operators give the difference between the total label carried by the fields minus the total label of the complex conjugated fields. We will also use a special notation which associates a momentum label index to an arbitrary product of collinear fields. Our convention is

$$\chi_{n,\omega} \equiv [W^\dagger \xi_n]_\omega = [\delta(\omega - \bar{n}\cdot\mathcal{P})W^\dagger\xi_n] \tag{3}$$

$$\left[W^\dagger iD_{\perp c}W\right]_\omega = [\delta(\omega - \bar{n}\cdot\mathcal{P})W^\dagger iD_{\perp c}W] \tag{4}$$

where $\delta(\omega - \bar{n}\cdot\mathcal{P})$ acts only inside the square brackets.

The collinear operators that we write in the following can include a nontrivial flavor structure. When required, this will be denoted by a superscript showing the quark flavours. Note, finally, that for each operator we can have the singlet and octet colour structure. We will write explicitly only the former since the matrix elements of any octet operator between the vacuum and a meson state vanishes.

At leading order in λ there are only three independent collinear operators, which can be chosen as

$$\mathcal{J}_V(\omega) = \bar{\chi}_{n,\omega_1}\frac{\bar{\not{n}}}{2}\chi_{n,\omega_2}\,, \tag{5}$$

$$\mathcal{J}_A(\omega) = \bar{\chi}_{n,\omega_1}\frac{\bar{\not{n}}}{2}\gamma_5\chi_{n,\omega_2}\,, \tag{6}$$

$$\mathcal{J}_T^\alpha(\omega) = \bar{\chi}_{n,\omega_1}\frac{\bar{\not{n}}}{2}\gamma_\perp^\alpha\chi_{n,\omega_2}\,, \tag{7}$$

where $\gamma_\perp^\alpha \equiv \gamma^\alpha - n^\alpha\bar{\not{n}}/2 - \bar{n}^\alpha\not{n}/2$ and $\omega = (\omega_1,\omega_2)$.

At subleading order in λ, we found the following four chiral-even collinear operators

$$\mathcal{V}_1^\alpha(\omega) = \left[\bar{\xi}_n\frac{\bar{\not{n}}}{2}(i\not{D}_{\perp c})^\dagger W_n\right]_{\omega_1}\frac{\gamma^\alpha}{\bar{n}\cdot\mathcal{P}^\dagger}\chi_{n,\omega_2} + \text{h.c.}\,, \tag{8}$$

$$\mathcal{V}_2^\alpha(\omega) = \left[\bar{\xi}_n\frac{\bar{\not{n}}}{2}(iD_{\perp c}^\alpha)^\dagger W_n\right]_{\omega_1}\frac{1}{\bar{n}\cdot\mathcal{P}^\dagger}\chi_{n,\omega_2} + \text{h.c.}\,, \tag{9}$$

$$\mathcal{A}_1^\alpha(\omega) = \left[\bar{\xi}_n\frac{\bar{\not{n}}}{2}(i\not{D}_{\perp c})^\dagger W_n\right]_{\omega_1}\frac{\gamma^\alpha\gamma_5}{\bar{n}\cdot\mathcal{P}^\dagger}\chi_{n,\omega_2} + \text{h.c.}\,, \tag{10}$$

$$\mathcal{A}_2^\alpha(\omega) = \left[\bar{\xi}_n\frac{\bar{\not{n}}}{2}(iD_{\perp c}^\alpha)^\dagger W_n\right]_{\omega_1}\frac{\gamma_5}{\bar{n}\cdot\mathcal{P}^\dagger}\chi_{n,\omega_2} - \text{h.c.}\,, \tag{11}$$

and the three chiral-odd operators

$$\mathcal{S}(\omega) = \left[\bar{\xi}_n\frac{\bar{\not{n}}}{2}(i\not{D}_{\perp c})^\dagger W_n\right]_{\omega_1}\frac{1}{\bar{n}\cdot\mathcal{P}^\dagger}\chi_{n,\omega_2} + \text{h.c.}\,, \tag{12}$$

$$\mathcal{P}(\omega) = \left[\bar{\xi}_n\frac{\bar{\not{n}}}{2}(i\not{D}_{\perp c})^\dagger W_n\right]_{\omega_1}\frac{\gamma_5}{\bar{n}\cdot\mathcal{P}^\dagger}\chi_{n,\omega_2} + \text{h.c.}\,, \tag{13}$$

$$\mathcal{T}^{\alpha\beta}(\omega) = \left[\bar{\xi}_n\frac{\bar{\not{n}}}{2}(iD_{\perp c}^\alpha)^\dagger W_n\right]_{\omega_1}\frac{\gamma_\perp^\beta}{\bar{n}\cdot\mathcal{P}^\dagger}\chi_{n,\omega_2} - \text{h.c.}\,, \tag{14}$$

together with the corresponding colour octet operators.

These operators are not the most general collinear gauge invariants at $O(\lambda)$; in fact, it is necessary to consider also the following three-particle operators

$$\mathcal{V}_3^\alpha(\omega) = \bar{\chi}_{n,\omega_1}\frac{\bar{\not{n}}}{2}\,[\frac{1}{\bar{n}\cdot\mathcal{P}}W^\dagger iD_\perp^\alpha W]_{\omega_3}\,\chi_{n,\omega_2} \tag{15}$$

$$\mathcal{A}_3^\alpha(\omega) = \bar{\chi}_{n,\omega_1}\frac{\bar{\not{n}}}{2}\gamma_5\,[\frac{1}{\bar{n}\cdot\mathcal{P}}W^\dagger iD_\perp^\alpha W]_{\omega_3}\,\chi_{n,\omega_2} \tag{16}$$

$$\mathcal{T}_3^{\alpha\beta}(\omega) = \bar{\chi}_{n,\omega_1}\frac{\bar{\not{n}}}{2}\gamma_\perp^\alpha\,[\frac{1}{\bar{n}\cdot\mathcal{P}}W^\dagger iD_\perp^\beta W]_{\omega_3}\,\chi_{n,\omega_2}\,. \tag{17}$$

In the definitions of the operators, (8)–(17), we inserted explicit $1/\bar{n}\cdot\mathcal{P}^{(\dagger)}$ factors to make them invariant under type-III reparameterization invariance (i.e. $n \to n\alpha$, $\bar{n} \to \bar{n}/\alpha$).

Let us finally discuss the way these operators appear in explicit calculations. A given QCD operator $\mathcal{O}_{\rm QCD}$ is matched onto SCET operators containing the subleading collinear bilinears introduced above

$$\mathcal{O}_{\rm QCD} = \cdots + \int \mathrm{d}\omega_1\mathrm{d}\omega_2 C_1(\omega_1,\omega_2)\{\cdots\}\mathcal{V}_i(\omega_1,\omega_2) \tag{18}$$
$$+ \int \mathrm{d}\omega_1\mathrm{d}\omega_2\mathrm{d}\omega_3 C_2(\omega_1,\omega_2,\omega_3)\{\cdots\}\mathcal{V}_i(\omega_1,\omega_2,\omega_3)$$

where the ellipses $\{\cdots\}$ denote possible soft fields which were omitted in writing the SCET operator. The Wilson coefficients $C_{1,2}(\omega_i)$ depend on the momentum labels of the collinear bilinears. After factorization, the matrix elements of the collinear operators $\mathcal{V}_i(\omega_i)$ between a light meson and the vacuum lead to non-perturbative functions $\langle M(p_M)|\mathcal{V}_i(\omega_1,\omega_2)|0\rangle \simeq \varphi_i(u)$, where we implemented momentum conservation $\omega_1 - \omega_2 = p_M$ by introducing the momentum fraction u by $(\omega_1,\omega_2) = (u,-\bar{u})\bar{n}\cdot p_M$, with $u \in [0,1]$. The charge-conjugation transformation properties of the collinear operators $\mathcal{V}_i(\omega_1,\omega_2)$, taken together with the C quantum number of the state $|M(p_M)\rangle$, fix the symmetry property of matrix elements under the substitution $u \to \bar{u}$. For example, taking $C = -1$ as appropriate for the ρ meson, one has

$$\langle\rho(p,\eta)|\mathcal{V}_i^{\rm even(odd)}(\omega_1,\omega_2)|0\rangle \sim \varphi_\rho^{\rm odd(even)}(u)\,, \tag{19}$$

such that only the odd (even) part of the corresponding Wilson coefficient $C(\omega_1,\omega_2)$ will give a non-vanishing contribution to the given matrix element of (18).

3 Matrix elements

In order to extract the matrix elements of the operators defined in the previous section, it is necessary project the five independent QCD currents onto $\mathrm{SCET_{II}}$. We refer to [14] for a detailed description of this procedure. Here we present only the projection of the vector current:

$$\bar{q}(x)\gamma^\mu[x,y]q(y) = \int \mathcal{D}^2\omega \left\{ \mathcal{J}_V(\omega)n^\mu + \mathcal{V}_1^\mu(\omega) + \frac{in^\mu}{2}\Big[(\omega_1\, x_\perp - \omega_2\, y_\perp)\cdot\mathcal{V}_2(\omega) + (\omega_1\, x_\perp + \omega_2\, y_\perp)\cdot\widetilde{\mathcal{V}}_2(\omega)\Big]\right\} -\frac{1}{2}\, n^\mu n\cdot z \int_0^1 \mathrm{d}t\,(tx_{\perp\alpha} + \bar{t}y_{\perp\alpha}) \int \mathcal{D}^3\omega\,\omega_3^2\,\mathcal{V}_3^\alpha(\omega)\,, \quad (20)$$

where $[x,y]$ is a gauge covariant Wilson line connecting $q(x)$ and $q(y)$ along a line. Using the explicit expressions for the vacuum-to-meson matrix elements of the QCD currents given in [9,10,11,12,13] we can extract the corresponding matrix elements for the SCET operators. Due to lack of space we present here only the results for a transverse polarized vector meson with momentum p and polarization $\eta_\perp$. The only non vanishing matrix elements are:

$$\langle V|\mathcal{J}_T^\mu|0\rangle = \frac{1}{4}\, f_V^T\,\eta_\perp^{\mu *}\,\bar{n}\cdot p\,\phi_\perp(u)\,, \quad (21)$$

$$\langle V|\mathcal{V}_1^\mu|0\rangle = f_V m_V \eta_\perp^{\mu *}\, g_\perp^{(v)}(u)\,, \quad (22)$$

$$\langle V|\mathcal{V}_2^\mu|0\rangle = -\frac{1}{2} f_V m_V \eta_\perp^{\mu *} \times \left[\frac{\bar{u}-u}{u\bar{u}}F(u) - \frac{G_{Vx}^{(v)}(u)}{u} - \frac{G_{Vy}^{(v)}(u)}{\bar{u}}\right], \quad (23)$$

$$\langle V|\mathcal{A}_1^\mu|0\rangle = \frac{i}{4} f_V m_V\,\epsilon_\perp^{\mu\nu}\eta_{\perp\nu}^*\, {g_\perp^{(a)}}'(u)\,, \quad (24)$$

$$\langle V|\mathcal{A}_2^\mu|0\rangle = \frac{i}{2} f_V m_V\,\epsilon_\perp^{\mu\nu}\eta_{\perp\nu}^* \times \left[\frac{\bar{u}-u}{u\bar{u}}\frac{g_\perp^{(a)}(u)}{4} - \frac{G_{Vx}^{(a)}(u)}{u} - \frac{G_{Vy}^{(a)}(u)}{\bar{u}}\right], \quad (25)$$

$$\langle V|\mathcal{V}_3^\mu|0\rangle = \frac{1}{2} f_V m_V\,\eta_\perp^{\mu *}\,\frac{\mathcal{V}(\alpha)}{\alpha_3^2}\,, \quad (26)$$

$$\langle V|\mathcal{A}_3^\mu|0\rangle = -\frac{i}{2} f_V m_V\,\epsilon_\perp^{\mu\nu}\eta_{\perp\nu}^*\,\frac{\mathcal{A}(\alpha)}{\alpha_3^2}\,, \quad (27)$$

where

$$G_{Vx}^{(v,a)}(u) = \frac{\mathrm{d}}{\mathrm{d}u}\int_0^u \mathrm{d}\alpha_1 \int_0^{\bar{u}} \mathrm{d}\alpha_2\,\frac{u-\alpha_1}{\alpha_3^2}(\mathcal{V},\mathcal{A})(\alpha)\,, \quad (28)$$

$$G_{Vy}^{(v,a)}(u) = \frac{\mathrm{d}}{\mathrm{d}u}\int_0^u \mathrm{d}\alpha_1 \int_0^{\bar{u}} \mathrm{d}\alpha_2\,\frac{\bar{u}-\alpha_2}{\alpha_3^2}(\mathcal{V},\mathcal{A})(\alpha) \quad (29)$$

and the various functions are defined in [9,10,11,12, 13]. Note that at leading order in $\Lambda_{\mathrm{QCD}}/m_b$ the matrix elements of the vector and axial structures vanish, $\langle V|\mathcal{J}_{V,A}^\mu|0\rangle = 0$ and only the tensor operator $\mathcal{J}_T^\mu$ has a non vanishing matrix element.

4 Reparameterization invariance

The soft-collinear effective theory has an additional symmetry, related to the Lorentz invariance of the full theory, which was explicitly broken by defining the effective theory in terms of the arbitrary light-cone vectors n_μ and $\bar{n}_\mu$. This symmetry manifests itself as an invariance under small changes in the light-cone vectors n_μ and $\bar{n}_\mu$, and is usually called reparameterization invariance (RPI). This invariance can be used to constrain the Wilson coefficients of the SCET operators introduced in Sect. 2.

Let us first point out that RPI of type III (invariance under rescaling of n and $\bar{n}$) implies that the SCET expansion must contain at least on additional vector in addition to n^μ and $\bar{n}^\mu$ [14]. This vector can be the heavy quark velocity v^μ or the space-time vector z^μ describing the non-locality of a T-product.

From the analysis presented in [14], it follows that the coefficients of the $O(\lambda)$ SCET operators appearing in the expansion of *any* QCD scalar current of the form $S(z)$ are completely determined in terms of the leading ones. The expansion of vector operators $V^\mu(z)$ is much more complicate and the coefficients of the $O(\lambda)$ operators are, nevertheless, severely constrained. These constraints are of no use for the case of the non-local vector current, (20), because the matching can be worked out using the equations of motion of QCD and is exact at all orders. On the other hand, in situations in which the coefficients have to be computed order by order in perturbation theory, RPI constraints provide a powerful tool to check and simplify the calculation.

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